Theme: **Suggested Days of Instruction: 45 days**

Ohio Learning Standards

NUMBER AND OPERATIONS: FRACTIONS (NF)

Use equivalent fractions as a strategy to add and subtract fractions.

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.
2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.
3. Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
   a. Interpret the product (a/b) x q as a parts of a partition of q into b equal parts; equivalently, as the results of a sequence of operations a x q ÷ b.
   b. Find the area of a rectangle with fractional sides lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction product as rectangular areas.
5. Interpret multiplication as scaling (resizing), by:
   a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
   b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater that the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (nxa)/(nxb) to the effect of multiplying a/b by 1.
6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
7. Apply & extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.
   a. Interpret division of a unit fraction by a non-zero whole number and compute such quotients.
   b. Interpret decision of a whole number by unit fractions, and compute such quotients.
   c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem

MEASUREMENT AND DATA (MD)

Represent and interpret data

2. Make a line plot to display a data set of measurements in fractions of a unit (1/2. ¼. 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots

NUMBER AND OPERATIONS IN BASE TEN (NBT)

Understand the base to system

1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns I the placement of the decimals point when decimals multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
3. Read, write and compare decimals to thousandths.
   a. Read and write decimals to thousandths using base-ten numerals, number names and expanded form, e.g., 347.392 = 3 x 100 + 4x10+7x1 +3 x(1/10) +9 x(1/100)+2x(1/1000).
   b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
4. Use place value understanding to round decimals to any place.

Perform operations with multi-digit whole numbers and with decimals to hundredths

5. Fluently multiply multi-digit whole numbers using the algorithm.
7. Add, subtract, multiply and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and / or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Commentary

Number and Operations in Fractions

In fourth grade, students added and subtracted fractions with like denominators and began to explore adding tenths plus hundredths in preparations for work with decimals. Students in Grade 5 extend this work to adding and subtracting fractions with unlike denominators using visual representations, reasoning, and equations.

Students worked with concrete models for multiplying a fraction by a whole number in Grade 4. They continue to extend this work to additional situations of multiplying a whole number by a fraction. They use area models to connect their understanding of multiplication of

Commentary

whole numbers to multiplication of fractions. Fifth graders make generalizations about multiplying fractions and whole numbers through using scaling as a model for multiplication and reasoning about the size of the product based on the size of the factors. The procedure for multiplying fractions is developed by making sense of what multiplication of fractions means rather than simply presenting students with a rule to follow. They solve a variety of multiplication problems applying their understanding to real-life situations.

Students explore division of a whole number by a fraction and a fraction by a whole number through visual models and contexts in order to make sense of what division fractions entails. They use concrete models and explain their reasoning as they work to apply previous understandings of division to fraction situations.

Measurement and Data

This cluster focuses on solving problems using line plots created to display measurement data in fractions of a unit.

Number and Operations in Base 10

Fifth Grade expand on previous work with place value to understand the relationship between adjacent places both to the left and to the right of a given place value. Connecting previous work with 10s, 100s and 1,000s to powers of ten gives students a sense of the magnitude of numbers and reinforces the relationship among place values.

Students begin to extend knowledge of place value to decimal numbers, including writing numbers to thousandths in expanded form and comparing decimals to the thousandths place.

In grade 5, students continue to add and subtract whole numbers with fluency. They apply previous experiences using models, strategies, plus value, and problem contexts in multiplication to an efficient algorithm. Student continue to work with various division examples and explore to find efficient procedures for division. (Note that use of a division algorithm is not expected until Grade 6.) Students extend their understanding of decimals to solve problems and calculation examples to add, subtract, multiply and divide decimals. They apply their understanding of the meaning of these operations from whole number experiences to using decimals.

Resources

Text: Math Expressions Common Core Assessment Guide

Unit 3: Multiplication with Fractions (Sections 3.1-3.14)

- Multiplication with Fractions
- Multiplication Links
- Division with Fractions

Unit 4: Multiplication with Whole Numbers & Decimals (Sections: 4.1-4.12)

- Multiplication with Whole Numbers
- Multiplication with Decimals Numbers

Source: Math Expressions: Common Core Assessment Guide

Differentiated Instruction Activities
Pages: 193, 199, 207, 217, 233, 241, 247, 253, 261, 267, 275, 281, 287, 303, 309, 315, 323, 327, 335, 345, 351, 359,367, 373 and 379

Think Central Resources: www.thinkcentral.com

Additional Resources
Below Level-Soar to Success, On Level-Mega Math, Challenge-Destination Math

Unit 3: Math Literature Library: "Go, Fractions!"
Unit 4: Math Literature Library: "The Amazing Impossible Erie Canal"
Technology: First In Math, Study Island, SchoolNet Short Cycle Assessments

Assessments

Source: Math Expressions: Common Core Assessment Guide
Unit 3 Quick Quiz 1,2 and 3 Unit 3 Assessment form A, Unit 3 Assessment form B
Unit 4 Quick Quiz 1 and 2 Unit 4 Assessment form A, Unit 4 Assessment form B

Adding Students Misconnections and Common Errors

5.NF.1

Watch for students who have surface understanding of the necessity for finding common denominators when adding and subtracting fractions and mixed numbers. Consistent practice in the forms of number task or using formative assessments task coupled with students explaining their thinking and considering the reasonableness of their solutions will help students to see the importance of thinking about of the numbers rather than using random calculations (add the numerators, add the denominators).

Relating the fractions to benchmark numbers (0, ½, 1) will help students to determine whether their answers are reasonable.

Two area that should be explicit in providing meaningful situations include considering the size of the pieces (that is, how many pieces make one whole or the denominator) and that the fractions must refer to the same size whole. Students must always consider the adding ½ of a small candy bar with ½ of a large candy bar will not produce 1 whole candy bar.

5.NF.2

Students who struggle to determine the appropriate operation to solve a problem need more experience with the problem situation for addition and subtraction. They need to use strategies such as act it out, draw a picture, write an equation, or make a model to determine how to best approach a problem. Give students opportunities to explain their thinking as they need the problems and use models to determine the correct operation. Make connections to earlier experiences with whole numbers that will help students to think of addition and subtraction in a particular situation. Once students determine the correction operation, they can use fractions and mixed numbers to solve the problem.

5.NF.3

Students may initially think that you cannot divide a “smaller number by a bigger number” since this will be a new situation for them to consider. Provide them with good problems to solve and give them many opportunities to explore with models so that they are developing conceptual understanding. It is important that they understand this concept in a way that makes sense to them rather than be shown how to do it. The role of the teacher is to provide sensible problem situations, ask supporting questions and facilitate conversations in which

Addressing Student Misconceptions and Common Errors Cont.

the students are making sense of the situation and why their answers make sense.

Students who struggle with interpreting the remainder of division examples need more experience solving problems using concrete models so they understand that the remainder tells what part of a group is left over. Asking questions such as "How many are left?" and "How many would it take to make another full group?" and modeling what part of a full group is left over will help them to understand the meaning of the remainder when it is expressed as a fraction.

5.NF.4

Students may see the pattern and see that to multiply fractions you "simply" multiply the numerators and multiply the denominators. This is the correct algorithm or procedure. However, only references to real-life situations and using models and visual representations will help students develop a conceptual understanding of what is actually happening they multiply fractions.

Watch for students who have difficulty determining the part of the unit square. Thinking in terms of the whole rectangle will help them define the number of parts when the dimensions are fractional parts of the whole. Reinforcing when they multiply a fraction by a fraction they are taking part of a part will help students to see that the "overlap" is the number of pieces (or numerator), and the total number of pieces in the whole is the denominator.

5.NF.5

Students will likely have many misconceptions about what happens to the product when one or both factors are scaled. For example, if both the length and width of a rectangle are doubled, some students will assume the product (area) is doubled. What they test their conjecture by drawing a picture, they will see that the product is actually for times greater. Allow students to explore a variety of multiplication scaling situations by drawing pictures and making models that will help them to make conjectures as to why the results are true, which is less likely to happen if they simply multiply.

Students are often puzzled when they find that the product is less than one or both of the factors. In previous work with multiplication of whole numbers, the product was always greater than both factors. Give students many opportunities to use visual models to "see" what is happening when they multiply with fractions. Discussions in which students explain their thinking will also help to identify and address misconceptions.

Addressing Student Misconceptions and Common Errors Cont.

5.NF.6

Watch for misconceptions from previous multiplication standards. Students who struggle understanding why they should multiply in these problems need more experience using visual representations. It is helpful to have them break the problem into smaller parts and explain their thinking as they complete each part of the problem.

5.NF.7

Dividing a fraction by a whole number is likely to cause students initial confusion around understanding how you can possibly divide a fraction (part of a whole) by a whole number. One misconception is that you always have to "divide the bigger number by the smaller number." Connect to work with earlier standards in this domain in which students interpreted a fraction such as ¾ to also mean 3 divided by 4. It is important to give students many opportunities to solve problems with visual representations to develop understanding that this is the same as the sharing situation they used when dividing whole numbers. Do not rush students into writing equations. Allow students to write their own problems modeled after those you have given. This will help them to thing about the situations and when it makes sense to divide a fraction by a whole number. Do not give them the tradition rule for division of fractions. Rather, take time for classroom discussions in which students explain their thinking and work to make sense out of the solution process and to determine the reasonableness of their answers. The role of the teacher is to clarify student thinking by posing good questions.

Watch for students who are having difficulty identifying what operation to use in solving problems with fractions. Using key words is not helpful and removes making sense from the process. Rather, have students model problem using pictures and asking supporting questions, such as "What do you know? What do you want to find out? How can you show that in your picture?" As students solved mixed numbers, adapt your questions to help students think about the meaning of the operations and how it can help them determine which operation to use.

Give students a variety of problems and ask them to model and write an expression they would use to solve the problem. Ask them to explain their model and expression.

Students may struggle determine which number goes where in the division problem. "Am I dividing the fraction by the whole number or the whole number by the fraction?" Drawing a picture using the information in the problem and focusing on what they want to find out will help. Model asking questions and encourage them to ask themselves similar questions, such as

Addressing Student Misconceptions and Common Errors Cont.

- What is being divided or broken up?
- Am I trying to determine how much in a group or how many groups?
- What visual representation can I use to show the actions of the problem?

5.MD.2

Some students may not know what measurement to use if the object measures between 1/8 and ¼ inch. To address this, help students understand that approximations can be used to measure to the closes 1/8 inch and ¼ inch.

Number and Operations in Base 10

5.NBT.1

Students get confused with the language describing the relationship between place value for whole numbers and decimals numbers. For example, when moving from ones to tens (one place to the left), the value is ten times greater, not ten greater or ten more. For example, when moving from tenths to hundredths (one place to the right), the value is 1/10 the value and not ten less than. Working with concrete models and pictorial representations and practicing with appropriate language will help students avoid confusion.

5.NBT.2

It is a major importance that students understand the relationship between the number of zeros in the power of ten, or the exponent and the number of zeros in the product beyond just noticing that they are the same. For example, when multiplying 62 X 100, students should be able to justify that the product represents 62 groups of 100, which is written as 6,200. This understanding is critical as students move to multiplying by decimals. The use of concrete materials and pictorial representations will help students to make these important connections.

As students begin to work with exponents they must understand that the exponent tells them the number of times the base (in this case 10) is used as a factor. Some students may think of addition (10^2 = 10 +10 instead of 10 X 10). Students need a variety of experiences including concrete and pictorial representations connected to skill practices to build understanding and skill working with exponential notation.

Addressing Student Misconceptions and Common Errors Cont.

5.NBT.3
It is imperative that students read decimals numbers correctly to reinforce the meaning of the decimal and its place value. For example, 1.12 should be read as "one and twelve hundredths" and not "one point twelve". Teachers should model this and the expectation should be clear. This not only reinforces the value of the decimal number but also explicitly connects decimal umber to fraction numbers.

When comparing two decimals with different place values, students may have the misconception that 0.345>0.57 because 345 os greater than 57 without considering the value of each place after the decimal point. Modeling these numbers using drawings or base-ten vlocks will help students to see that 0.57 is greater because there are ore tenths. They should have many opportunities to use models and drawings and explain their thinking. Once students demonstrate understanding using concrete materials, they should begin to explore why 0.57 is equivalent to 0.57 using models, place value understanding and previous experience with equivalent fractions. If both decimal numbers have the same number of places, thousandths for example, students can then compare the number of thousandths in each decimal number to determine which is greater and which less is.

5.NBT.4
Students who are taught to round decimals by using a rule rather than place value understanding have difficulty determining places when rounding up or down. This is true with both whole numbers and decimals. For example, when rounding to the nearest tenth, a student might round 15.28 to 15.38. When using a number line model, students need to determine the number that the given number falls between. In the previous case it would be between 15.2 and 15.3. Using benchmark numbers such as 15.25, which falls exactly in the middle, will help students determine the closest tenth. By plotting the given point on the number line, students can determine to which tenth it is closer. Scaffold examples for students who are struggling with this concept.

5.NBT.5
Students who become confused with regrouping in multi-digit multiplication need additional experiences with the partial
Product algorithm. Once they are proficient multiplying using partial products, they can begin to consider how using regrouping can save several steps. Scaffold examples for these students and give them time to understand how both the partial product and the regrouping algorithm are alike.

Addressing Student Misconceptions and Common Errors Cont.

5NBT.7

Student misconceptions when working with decimals number are usually based on place value. Simply telling students to line up the decimal points when adding and subtracting decimals does not build the important understanding that similar place values are to be adding and subtracting decimals does not build the important understanding that similar place values are to be added or subtracted and can lead to errors such as the following:

$$\begin{array}{r} 2.5 \\ \underline{-1.75} \\ 0.85 \end{array}$$

Build on the whole-number using concrete materials and place value charts will help students to relate previous work with composing and decomposing whole numbers to composing and decomposing decimals.

Since the "rules" for multiplication and division of decimals are much easier to teach than developing place value understanding we are often tempted to provide students with these rules at some point. Don't do it! Students need time to see the structure of multiplication and division of decimals and how it relates to whole-number multiplication and division. Without this foundation, students may move decimal points when it is convenient rather than when it is necessary. Place value understanding also allows students to determine whether answers are reasonable. It is far more meaningful to students when they can generalize rules after many experiences and good questions from the teacher.

Source: The Common Core Mathematics Companion: The Standards Decoded (What They Say, What They Mean, How to Teach Them)
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