Theme 1: The Number Systems **Suggested Days of Instruction: 25 days**

Ohio's Learning Standards

NUMBER SYSTEMS (NS)

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

a. Describe situations in which opposite quantities combine to make 0.
b. Understand p + q as the number located a distance $|q|$ from p, in the positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
c. Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (-q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
d. Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiply signed numbers. Interpret products of rational numbers by describing real-world contexts.
b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-\left({}^{p}/_{q}\right) = {}^{(-p)}/_{q} = {}^{p}/_{(-q)}$. Interpret quotients of rational numbers by describing real-world contexts.
c. Apply properties of operations as strategies to multiply and divide rational numbers.
d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Commentary:
This unit is about understanding and computing with rational numbers. Rational numbers include integers, positive and negative fractions, and positive and negative decimals. Students learn how to add, subtract, multiply, and divide integers and apply properties of operations as strategies for each operation. Students journey from exploring the operations to formalizing rules. Students convert rational numbers to decimal form using division. The understanding of a rational numbers as one that terminates or repeats is covered in Grade 7 as preparation for the introduction of irrational numbers in Grade 8.

Resources:
SpringBoard: Unit 1 (Lessons 1-1 to 4 - 4)
Tools: Number line, Two-Color counters, Algebra Tiles

Formative Assessments
SpringBoard Digital: Short-Cycle Assessment for each lesson

Summative Assessments
SpringBoard Digital: End of Unit or Customized Assessment

OST Released Items:

Embedded Assessments
Unit 1
EA1: Positive Rational Numbers and Adding and Subtracting, ***Off to the Races***
EA2: Rational Number Operations and Multiplying and Dividing Integers, ***Top to Bottom***

Addressing Student Misconceptions and Common Errors
7.NS.1
Students may understand that one positive and one negative make zero but have difficulty understanding that this is also true for all equal amounts of positives and negatives such as five positives and five negatives. One way to make his clear is to start with one positive and one negative counter. As soon as the student establishes that this zero, add another pair. When the student recognizes that you have just added another zero to the first zero, repeat. Repeat until the student has developed the concept.

Students who are not able to solve equations abstractly as quickly as others may need to use number lines and/or two-color counters for a longer period of time until they understand the concepts. Algebra Tiles can also be used as models.

In previous grades students learned that subtraction is not commutative. This holds true with rational numbers even though students now understand that 6 – 8 = (-2). It is still the case that $6 - 8 \neq 8 - 6$.

7.NS.2
Students who are not fluent with basic multiplication and division facts will have difficulty performing fluently with rational numbers. These students need additional practice. Many computer-based programs have success with basic fact mastery.

Addressing Student Misconceptions and Common Errors - Continued

7.NS.2

Students who are not fluent with basic multiplication and division facts will have difficulty performing fluently with rational numbers. These students need additional practice. Many computer-based programs have success with basic fact mastery. For students who do not understand why division by 0 is undefined, give specific examples of the relationship between multiplication and division that would not make sense. For example, $x \times 0 = 5, so\ 5 \div 0 = x$. There is no possible number for x.

Students often confuse the associative property with the distributive because both properties use parentheses. This is common error for students who do not understand what is happening in the equation. Try modeling the properties with simple examples such as this one for the distributive property: I have 3 tetras and 4 goldfish in each of my 5 fish tanks. How many fish do I have?

$$5(3 + 4) = (5 \times 3) + (5 \times 4)$$

Some students may have difficulty with long division. Look for patterns in repeated errors made by these students and target these specific errors. Common division errors are forgetting to add zeros in the dividend and placing the decimal point in the wrong place in the quotient. Try some alternate algorithms that focus on place value. Some students may have tracking difficulties. Doing long division on graph paper is a solution for many students.

Perseverance is important in converting to decimal form if there are many digits that repeat. Students should be encouraged to continue until they find a repeating pattern or 0 because it is a common error that they stop the division algorithm too soon. Calculators are a useful tool for decimals that have four digits that repeat.

Sometimes students want to use the repeat bar over two or three repeating digits when only one digit repeats such as $\frac{3}{9} = 0.\overline{3}$, not $0.\overline{333}$. This error can be corrected through the use of error analysis exercises where students are presented with incorrect work done by a fictitious student. The problems should re-create a common error or misconception and the students try to identify it, thereby clarifying their own thinking. In this case, an example is as follows: Mario wrote the answer to a problem as $\frac{1}{3} = 0.\overline{333}$. He made an error. Correct his mistake and explain to Mario how to avoid making the same mistake in the future.

7.NS.3

As equations become longer with more terms and more complex using rational numbers, some students are overwhelmed and do not know where to begin. Help these students by reviewing the order of operations and demonstrating how to solve equations one step at a time. Flip books created by the students that do a step-by-step breakdown of a computation aid some students. For such a book, students can begin with a problem and perform one step on the first page, then repeat that step and add a second step to the next page, continuing in this matter.

Source: The Common Core Mathematics Companion: The Standards Decoded (What They Say, What They Mean, How to Teach Them)
Authors: Ruth Harbin Miles and Lois A. Williams, 2016 NCTM

Theme 2: Expressions and Equations **Suggested Days of Instruction: 15 days**

Ohio's Learning Standards

EXPRESSIONS AND EQUATIONS (EE)

Use properties of operations to generate equivalent expressions.

7.EE.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.2 In a problem context, understand that rewriting an expression in an equivalent form can reveal and explain properties of the quantities represented by the expression and can reveal how those quantities are related.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimations strategies.

7.EE.4 Use variables to represent quantities in real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Commentary:

Seventh graders use properties of operations to generate equivalent expressions. They use the arithmetic of rational numbers to formulate expressions and equation in one variable and use these equations to solve problems. The seventh-grade focus of solving real-world and mathematical problems using numerical and algebraic expressions and equations provides the foundation for equation work for writing equivalent nonlinear expressions in later grades.

Resources:
SpringBoard: Unit 2 (Lessons 5-1 to 7 - 2)
Tools: Color Tiles, Graph Paper

Embedded Assessments
Unit 1
EA1: Writing and Solving Equations, ***Fundraising Fun***
EA2: Solving Inequalities, ***A Gold Medal Appetite***

Formative Assessments
SpringBoard Digital: Short-Cycle Assessment for each lesson

Summative Assessments
SpringBoard Digital: End of Unit or Customized Assessment

OST Released Items:

Addressing Student Misconceptions and Common Errors
7.EE.1
When students work with several steps in an expression, sometimes they forget about the order of operations such as in the following example: 7 + 2(3x – 5) + 2x. Students may want to add the 7 + 2 first or only multiply the 2 by the 3x and not the -5. A review of the order of operations can help. For students who need more assistance, have them create their own order of operations card with steps outlined to reference when needed to check their work. Students can also create their own pneumonic device to help them recall the steps.

7.EE.2
Many students have difficulty seeing that expressions are equivalent when the expressions are out of context. Use simple contexts so that students can reason with a context to explain why two expressions are equivalent. For example: "Write two equivalent expressions for the following situation – All music downloads are 99 cents today. Maria wants to download 2 R7B hits, I rap hit, and 3 hits by her favorite artist. Two equivalent expressions are 6 x 0.99 and (2 x 0.99) + (1 x 0.99) + (3 x 0.99)." Focus student attention on how 6 hits for 0.99 each is the same as 2 hits and 1 hit and 3

Addressing Student Misconceptions and Common Errors – Cont.

7.EE.3

It is common for students to have difficulty with multi-step problems. Scaffold the problems by adding a question mid-way. Display the first step of the problem, allow students to find the answer, and then present the next part that relies on the first step. Gradually remove the middle question as students get used to finding a middle question and identifying it themselves. For example: "Fred goes out to eat and buys a pizza that costs $12.75, including $.50 tax. He wants to leave a tip based on the cost of the food. What must Fred do?"

First, present the following: "Fred goes out to eat and buys a pizza that costs $12.75, including $.50 tax. How much did the pizza cost?" Solve this part of the problem. Then, using the answer from Part 1, introduce the second part of the problem: He wants to leave a tip based on the cost of the food. What Fred do?"

Some students' work may indicate a weakness representing numbers in different forms such as 10% as $\frac{1}{10}$. These students need additional practice. Use number lines, visuals such as bars, and hand-on materials instead of memorizing rules.

7.EE.4

Students who have difficulty becoming fluent in solving equations may need a hands-on approach. Manipualtives such as Algeblocks, Hands-On Equations and Algebra Tiles can be useful.

Students may forget to switch the inequality sign when multiplying or dividing by a negative. Help students by asking them to check answers in their solution sets in the original inequality to see if they satisfy the inequality. For other students who consistently make errors, check their number line graphs. Some seventh graders may have difficulty drawing the graphs accurately. For example, some students will reverse the location of negative and positive integers. For these students, supply them with graph paper or simply a sheet of pre-drawn number lines for them to fill out.

Source: The Common Core Mathematics Companion: The Standards Decoded (What They Say, What They Mean, How to Teach Them)
Authors: Ruth Harbin Miles and Lois A. Williams, 2016 NCTM