**Theme:** **Suggested Days of Instruction: 45 days**

**Ohio Learning Standards**

**NUMBER AND OPERATIONS: FRACTIONS (NF)**

***Use equivalent fractions as a strategy to add and subtract fractions.***

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.
2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.

***Apply and extend previous understandings of multiplication and division to multiply and divide fractions.***

3. Interpret a fraction as division of the numerator by the denominator (a/b = a ÷b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
4. Apply and extend previous understanding of multiplication to multiply a fraction or whole number by a fraction.
   a. Interpret the product (a/b) x q as a parts of a partition of q into b equal parts; equivalently, as the results of a sequence of operations a x q ÷ b.
   b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
5. Interpret multiplication as scaling (resizing), by:
   a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
6. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.
7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.
   a. Interpret division of a unit fraction by a non-zero whole number and compute such quotients.
   b. Interpret decision of a whole number by unit fractions, and compute such quotients.
   c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.

**OPERATIONS AND ALGEBRAIC THINKING (OA)**

***Write and interpret numerical expressions.***

1. Use parentheses brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

**NUMBER AND OPERATIONS IN BASE TEN (NBT)**

***Understand the base to system***

2. Explain patterns in the number of zeros of the product when

multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when decimals multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
3. Read, write and compare decimals to thousandths.
b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
4. Use place value understanding to round decimals to any place.

***Perform operations with multi-digit whole numbers and with decimals to hundredths***

5. Fluently multiply multi-digit whole numbers using the algorithm
6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and /or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
7. Add, subtract, multiply and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and / or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

## Commentary

Number and Operations in Fractions

Fifth graders build on previous experiences with fractions and use a variety of visual models and strategies to add and subtract fractions and mixed numbers with unlike denominators. Problem solving provides context for students to use mathematical reasoning to determine whether their answers make sense. They extend their understanding of fractions as parts of a whole to interpret a fraction as a division representation of the numerator divided by the denominator. Students use this understanding in the context of dividing whole numbers with an answer in the form of a fraction or mixed number. They continue to build conceptual understanding of multiplication of fractions using visual models and connecting the meaning to the meaning of multiplication of whole numbers. The meaning of the operation is the same, however, the procedure is different. Students use visual models and problem-solving context to develop understanding of dividing a unit fraction by a whole number and a whole number by a unit fraction. Once conceptual understanding is established, students generalize efficient procedures for multiplying and dividing fractions.

**Cluster A: Use equivalent fractions as a a strategy to add and subtract fractions.**

**Commentary cont.**

In fourth grade students added and subtracted fractions with like denominators and began to explore adding tenths plus hundredths in preparation for work with decimals. Students in Grade 5 extend this work to adding and subtracting fractions with unlike denominators using visual representations, reasoning, and equations.

**Standards for Mathematical Practices**

**SFMP 1. Make sense of problems and persevere in solving them.**

**SFMP 2. Use quantitative reasoning.**

**SFMP 3. Construct viable arguments and critique the reasoning of others.**

**SFMP 4. Model with Mathematics.**

**SFMP 5. Use appropriate tools strategically.**

**SFMP 6. Attend to precision.**

**SFMP 7. Look for and make use of structure.**

**SFMP 8. Look for and express regularity in repeated reasoning.**

Problem solving provides the context students use to develop conceptual understanding of addition and subtraction of fractions and mixed numbers. Students use quantitative reasoning to determine whether their answers make sense. Because a common error when adding or subtracting fractions with unlike denominators is to add or subtract the numerators and denominators, using benchmark fractions to reason about the value of the fractions will help students realize that if they add or subtract denominators their answer will not be reasonable. Using appropriate models including area modes, fractions bars, and the number line will help students to develop efficient strategies for adding and subtracting fractions and mixed numbers.

**Cluster B: Apply and extend previous understanding of multiplication and division to multiply and divide fractions.**

Students worked with concrete models for multiplying a fraction by a whole number in Grade 4. They continue to extend this work to additional situations of multiplying a whole number by a fraction. They use areas models to connect their understanding of multiplication of whole number to multiplication of whole numbers through using scaling as a model for multiplication and reasoning about the size of the

**Commentary cont.**

factors. The procedure for multiplying fractions is developed by making sense of what multiplication of fractions means rather than simply presenting students with a rule to follow. They solve a variety of multiplication problems applying their understanding to real-life situations.

Students explore division of a whole number by a fraction and a fraction by a whole number through visual models and contexts in order t make sense of what division fractions entails. They use concrete models and explain their reason as they work to apply previous understanding of division to fraction situations.

**Standards for Mathematical Practices**

**SFMP 1. Make sense of problems and persevere in solving them.**

**SFMP 2. Use quantitative reasoning.**

**SFMP 3. Construct viable arguments and critique the reasoning of others.**

**SFMP 4. Model with Mathematics.**

**SFMP 5. Use appropriate tools strategically.**

**SFMP 6. Attend to precision.**

**SFMP 7. Look for and make use of structure.**

**SFMP 8. Look for and express regularity in repeated reasoning.**

Students use a variety of problem solving situations to develop understanding of multiplication of fractions and mixed numbers. They solve and write problems that include dividing a fraction by a whole number and a whole number by a fraction using models and verbal explanations. They make connections between the structure of whole number multiplication and division models and how the meaning of these operations is the same with whole numbers and fractions but the actual procedures are quite different. Following many opportunities to model, explain and solve problems, students use their experiences to recognize patterns and develop efficient strategies.

**Operations and Algebraic Thinking**

In preparation for the Expressions and Equations domain in Grades 6 -8, fifth graders begin to explore, interpret, and evaluate numerical expressions. Work with patterns that began in Grade 4 extends to generating patterns, forming ordered pairs, graphing on a coordinate

**Commentary Cont.**

plane, and then analyzing the graphical representations.

**Cluster A: Write and interpret numerical expressions.**

In grade 5, students continue to explore and work with numerical expression in preparation for the Expressions and Equations domain coming in middle school. Students worked informally with order of operations in grades 3 and 4 as they solved multi-step problems through modeling and writing equations. According to the standards progression document (http://ime.math.arizona.edu/origressions/), this work should be exploratory, and expressions need not include nestling symbols.

**Standards for Mathematical Practices**

**SFMP 3. Construct viable arguments and critique the reasoning of others.**

**SFMP 5. Use appropriate tools strategically.**

**SFMP 6. Attend to precision.**

**SFMP 7. Look for and make use of structure.**

**SFMP 8. Look for and express regularity in repeated reasoning.**

As students apply rules for order of operations they should explain their reasoning to others. They use scientific calculators (those that follow order of operations) to explore order of operations. Students use appropriate vocabulary to derive their work with grouping symbols and order of operations. As students explore order of operations and apply the rule in a variety of situations, they look for patterns and the structure of what is happening. They understand d and apply calculating all multiplications and divisions before additions and subtractions within an expression. They make generalizations about the order of operations and grouping symbols and apply these rules to writing and solving expressions that include more than one operation and or grouping symbols.

**Number and Operations in Base Ten**

Fifth graders extend their work with place value to include decimals numbers to the thousandths place. They use efficient algorithms to multiply multi-digit whole numbers. They begin to divide whole numbers with two-digit divisors. They extend their understanding g of whole number operations to adding, subtraction, multiplying and dividing decimals to hundredths.

**Commentary Cont.**

The domain is not taught in isolation from the Operations and Algebraic Thinking domain. Students work across domains to develop a deep understanding of addition and subtraction by focusing on the instructional shift of rigor that is, developing conceptual understanding, building skill and fluency, and applying all four operations in problem contexts.

**Cluster A: Understand the place value system**

Fifth graders expand on previous work with place value to understand the relationship between adjacent places both to the left and the right of a given place value. Connecting previous work with 10s, 100s, 1,00s to powers of ten fives students a sense of magnitude of number and reinforces the relationship among place values.

Students begin to extend knowledge of place value to decimal numbers, including writing numbers to the thousandths in expanded form comparing decimals to the thousandths place.

**Standards for Mathematical Practices**

**SFMP 2. Use quantitative reasoning.**

**SFMP 3. Construct viable arguments and critique the reasoning of others.**

**SFMP 4. Model with Mathematics.**

**SFMP 5. Use appropriate tools strategically.**

**SFMP 6. Attend to precision.**

**SFMP 7. Look for and make use of structure.**

This cluster contains various aspects of using place value with both whole numbers and decimals numbers. Different mathematical practices help students develop understanding and apply the mathematics in each Standard. Using models and quantitative reasoning will help students to understand the relationship between adjacent places in both whole numbers and decimals and reinforce conceptual understanding of individual places as well as the magnitude of a number across place values on both sides of the decimal point. The use of exponents in expressing powers of ten is new to this grade level, and precision in writing and explaining powers of ten is critical. Students also connect this new written notation with exponents to the structure of our place value system.

**Commentary cont.**

**Cluster B: Perform operations with multi-digit whole number and with decimals to hundredths.**

In grade 5, students continue to add and subtract whole numbers with fluency. They apply previous experiences using models, strategies, place value and problem contexts in multiplication to an efficient algorithm. Students continue to work with various division examples and explore to find efficient procedures for division (Note that use a division algorithm is not expected until Grade 6.) Students extend their understanding of decimals to solve problems and calculation examples to add, subtract, multiply, and divide decimals. They apply their understanding of the meaning of these operations from whole number experiences to using decimals.

**Standards for Mathematical Practices**

**SFMP 1. Make sense of problems and persevere in solving them.**

**SFMP 2. Use quantitative reasoning.**

**SFMP 3. Construct viable arguments and critique the reasoning of others.**

**SFMP 4. Model with Mathematics.**

**SFMP 5. Use appropriate tools strategically.**

**SFMP 6. Attend to precision.**

**SFMP 7. Look for and make use of structure.**

**SFMP 8. Look for and express regularity in repeated reasoning.**

Although this cluster does not explicitly address expecting students to solve problems, providing contexts for computation will support students in extending the meaning of multiplication and division from whole numbers to operations with decimals. Students think quantitatively as they apply earlier strategies to a standard algorithm in multiplication. Models and tools, including area models, the number line, and partial products, will help to connect conceptual understanding to procedurally skills. Students explain their thinking using precise vocabulary. As they move from previous multiplication strategies to using an efficient algorithm, students are making use of the structure of mathematics, including the use of place value and properties.

**Resources**

**Text: Math Expressions Common Core Assessment Guide**

**Unit 5: Division with Whole Numbers (Sections 5.1-5.11)**
- Division with Whole Numbers
- Division with Decimals Numbers

**Unit 6 Operations and World Problems (Sections: 6.1-6.11)**
- Equations and Problem Solving
- Comparison Word Problems
- Problems with More Than One Step

**DIFFERENTION**

**Source: Math Expressions: Common Core Assessment Guide**

**Differentiated Instruction Activities**
Pages: 395, 401, 407, 415, 421, 429, 439, 447, 453, 461, 465, 479, 487, 493, 501, 509, 517, 525, 533, 539, 545 and 551

**Think Central Resources: www.thinkcentral.com**
Below Level-Soar to Success, On Level-Mega Math, Challenge-Destination Math

**Assessments**
**Source: Math Expressions: Common Core Assessment Guide**

Unit 5 Quick Quiz 1 and 2 Unit 5 Assessment form A, Unit 5 Assessment form B
Unit 6 Quick Quiz 1, 2 and 3 Unit 6 Assessment form A, Unit 6 Assessment form B

**Materials:**

**Unit 5:** Math Journals; activity cards; play money; calculator; index cards; grocery store ads; small 10x10 grids; colored pencils; sharpened pencils; metric ruler; scissors

**Unit 6:** Activity Cards; Math journals

**Adding Students Misconnections and Common Errors**

**5.NF.1**

Watch for students who have surface understanding of the necessity for finding common denominators when adding and subtracting fractions and mixed numbers. Consistent practice in the forms of number task or using formative assessments task coupled with students explaining their thinking and considering the reasonableness of their solutions will help students to see the importance of thinking about of the numbers rather than using random calculations (add the numerators, add the denominators).

Relating the fractions to benchmark numbers (0, ½, 1) will help students to determine whether their answers are reasonable.

Two area that should be explicit in providing meaningful situations include considering the size of the pieces (that is, how many pieces make one whole or the denominator) and that the fractions must refer to the same size whole. Students must always consider the adding ½ of a small candy bar with ½ of a large candy bar will not produce 1 whole candy bar.

**5.NF.2**

Students who struggle to determine the appropriate operation to solve a problem need more experience with the problem situation for addition and subtraction. They need to use strategies such as act it out, draw a picture, write an equation, or make a model to determine how to best approach a problem. Give students opportunities to explain their thinking as they need the problems and use models to determine the correct operation. Make connections to earlier experiences with whole numbers that will help students to think of addition and subtraction in a particular situation. Once students determine the correction operation, they can use fractions and mixed numbers to solve the problem.

**5.NF.3**

Students may initially think that you cannot divide a “smaller number by a bigger number” since this will be a new situation for them to consider. Provide them with good problems to solve and give them many opportunities to explore with models so that they are developing conceptual understanding. It is important that they understand this concept in a way that makes sense to them rather than be shown how to do it. The role of the teacher is to provide sensible problem situations, ask supporting questions and facilitate conversations in which the students are making sense of the situation and why their answers make sense.

**Addressing Student Misconceptions and Common Errors Cont.**

Students who struggle with interpreting the remainder of division examples need more experience solving problems using concrete models so they understand that the remainder tells what part of a group is left over. Asking questions such as "How many are left?" and "How many would it take to make another full group?" and modeling what part of a full group is left over will help them to understand the meaning of the remainder when it is expressed as a fraction.

**5.NF.4**

Students may see the pattern and see that to multiply fractions you "simply" multiply the numerators and multiply the denominators. This is the correct algorithm or procedure. However, only references to real-life situations and using models and visual representations will help students develop a conceptual understanding of what is actually happening they multiply fractions.

Watch for students who have difficulty determining the part of the unit square. Thinking in terms of the whole rectangle will help them define the number of parts when the dimensions are fractional parts of the whole. Reinforcing when they multiply a fraction by a fraction they are taking part of a part will help students to see that the "overlap" is the number of pieces (or numerator), and the total number of pieces in the whole is the denominator.

**5.NF.5**

Students will likely have many misconceptions about what happens to the product when one or both factors are scaled. For example, if both the length and width of a rectangle are doubled, some students will assume the product (area) is doubled. What they test their conjecture by drawing a picture, they will see that the product is actually for times greater. Allow students to explore a variety of multiplication scaling situations by drawing pictures and making models that will help them to make conjectures as to why the results are true, which is less likely to happen if they simply multiply.

Students are often puzzled when they find that the product is less than one or both of the factors. In previous work with multiplication of whole numbers, the product was always greater than both factors. Give students many opportunities to use visual models to "see" what is happening when they multiply with fractions. Discussions in which students explain their thinking will also help to identify and address misconceptions.

**5.NF.6**

Watch for misconceptions from previous multiplication standards. Students who struggle understanding why they should multiply in these

**Addressing Student Misconceptions and Common Errors Cont.**

problems need more experience using visual representations. It is helpful to have them break the problem into smaller parts and explain their thinking as they complete each part of the problem.

**5.NF.7**

Dividing a fraction by a whole number is likely to cause students initial confusion around understanding how you can possibly divide a fraction (part of a whole) by a whole number. One misconception is that you always have to "divide the bigger number by the smaller number." Connect to work with earlier standards in this domain in which students interpreted a fraction such as ¾ to also mean 3 divided by 4. It is important to give students many opportunities to solve problems with visual representations to develop understanding that this is the same as the sharing situation they used when dividing whole numbers. Do not rush students into writing equations. Allow students to write their own problems modeled after those you have given. This will help them to thing about the situations and when it makes sense to divide a fraction by a whole number. Do not give them the tradition rule for division of fractions. Rather, take time for classroom discussions in which students explain their thinking and work to make sense out of the solution process and to determine the reasonableness of their answers. The role of the teacher is to clarify student thinking by posing good questions.

Watch for students who are having difficulty identifying what operation to use in solving problems with fractions. Using key words is not helpful and removes making sense from the process. Rather, have students model problem using pictures and asking supporting questions, such as "What do you know? What do you want to find out? How can you show that in your picture?" As students solved mixed numbers, adapt your questions to help students think about the meaning of the operations and how it can help them determine which operation to use.

Give students a variety of problems and ask them to model and write an expression they would use to solve the problem. Ask them to explain their model and expression.

Students may struggle determine which number goes where in the division problem. "Am I dividing the fraction by the whole number or the whole number by the fraction?" Drawing a picture using the information in the problem and focusing on what they want to find out will help. Model asking questions and encourage them to ask themselves similar questions, such as:

- What is being divided or broken up?
- Am I trying to determine how much in a group or how many groups?
- What visual representation can I use to show the actions of the problem?

**Addressing Student Misconceptions and Common Errors Cont.**

**5.OA.1**

Some students will have the misconception that all multiplications are calculated before divisions and additions are calculated before subtractions. Scaffold examples for students to practice solving multiplications and/divisions in order from left to right and then additions and/or subtractions in order from left to right. Although parentheses are not necessary when the equation is written accurately, some students will find it helpful to add grouping symbols in order to solve equations and word problems.

**5.NBT.2**

It is a major importance that students understand the relationship between the number of zeros in the power of ten, or the exponent and the number of zeros in the product beyond just noticing that they are the same. For example, when multiplying 62 X 100, students should be able to justify that the product represents 62 groups of 100, which is written as 6,200. This understanding is critical as students move to multiplying by decimals. The use of concrete materials and pictorial representations will help students to make these important connections.

As students begin to work with exponents they must understand that the exponent tells them the number of times the base (in this case 10) is used as a factor. Some students may think of addition ($10^2$ = 10 +10 instead of 10 X 10). Students need a variety of experiences including concrete and pictorial representations connected to skill practices to build understanding and skill working with exponential notation.

**5.NBT.3**

It is imperative that students read decimals numbers correctly to reinforce the meaning of the decimal and its place value. For example, 1.12 should be read as “one and twelve hundredths” and not “one point twelve”. Teachers should model this and the expectation should be clear. This not only reinforces the value of the decimal number but also explicitly connects decimal umber to fraction numbers.

When comparing two decimals with different place values, students may have the misconception that 0.345>0.57 because 345 of greater than 57 without considering the value of each place after the decimal point. Modeling these numbers using drawings or base-ten blocks will help students to see that 0.57 is greater because there are ore tenths. They should have many opportunities to use models and drawings and explain their thinking. Once students demonstrate understanding using concrete materials, they should begin to explore why 0.57 is equivalent to 0.57 using models, place value understanding and previous experience with equivalent fractions. If both decimal numbers have the same number of places, thousandths for example, students can then compare the number of thousandths in each decimal number to determine which is greater and which less is.

**5.NBT.4**

Students who are taught to round decimals by using a rule rather than place value understanding have difficulty determining places when

**Addressing Student Misconceptions and Common Errors Cont.**

rounding up or down. This is true with both whole numbers and decimals. For example, when rounding to the nearest tenth, a student might round 15.28 to 15.38. When using a number line model, students need to determine the number that the given number falls between. In the previous case it would be between 15.2 and 15.3. Using benchmark numbers such as 15.25, which falls exactly in the middle, will help students determine the closest tenth. By plotting the given point on the number line, students can determine to which tenth it is closer. Scaffold examples for students who are struggling with this concept.

**5.NBT.5**

Students who become confused with regrouping in multi-digit multiplication need additional experiences with the partial product algorithm. Once they are proficient multiplying using partial products, they can begin to consider how using regrouping can save several steps. Scaffold examples for these students and give them time to understand how both the partial product and the regrouping algorithm are alike.

**5.NBT.6**

Division is a complex operation, and students who depend on following rote steps cannot determine whether their answer is reasonable. Emphasis on place value and connections to multiplication will help students to develop a deeper understanding of division. All division experiences should be developed in the context of asking questions such as "How Many groups of 20 can you make from 700?" and them allowing students to estimate and identify the number of objects (for example, If I make 30 groups of 20 that would be 600, and if I make 40 groups that would be 800, and that is too high). Such reasoning will help students to home in on a good estimate and use partial products to determine the exact quotient.
Students may still need additional experiences with the meaning of the remainder built on previous work in Grade 4. Problems in which the remainder is the answer, in which the remainder is dropped, or in which the quotient should be one more because of a remainder should all be included in division problems students are asked to solve.

**5NBT.7**
Student misconceptions when working with decimals number are usually based on place value. Simply telling students to line up the decimal points when adding and subtracting decimals does not build the important understanding that similar place values are to be adding and subtracting decimals does not build the important understanding that similar place values are to be added or subtracted and can lead to errors such as the following:

**Addressing Student Misconceptions and Common Errors Cont.**

$$\begin{array}{r} 2.5 \\ \underline{-1.75} \\ 0.85 \end{array}$$

Build on the whole-number using concrete materials and place value charts will help students to relate previous work with composing and decomposing whole numbers to composing and decomposing decimals.
Since the "rules" for multiplication and division of decimals are much easier to teach than developing place value understanding we are often tempted to provide students with these rules at some point. Don't do it! Students need time to see the structure of multiplication and division of decimals and how it relates to whole-number multiplication and division. Without this foundation, students may move decimal points when it is convenient rather than when it is necessary. Place value understanding also allows students to determine whether answers are reasonable. It is far more meaningful to students when they can generalize rules after many experiences and good questions from the teacher.

Source: The Common Core Mathematics Companion: The Standards Decoded (What They Say, What They Mean, How to Teach Them)
Authors: Ruth Harbin Miles and Lois A. Williams, 2016 NCTM