

THEME: PLACE VALUE AND MULTI-DIGIT ADDITION AND SUBTRACTION**OPERATIONS AND ALGEBRAIC (OA)****Represent and solve problems involving multiplication and division.**

- 3.OA.1 Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. (Note: These standards are written with the convention that $a \times b$ means a groups of b objects each; however, because of the commutative property, students may also interpret 5×7 as the total number of objects in 7 groups of 5 objects).
- 3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or number of groups can be expressed as $56 \div 8$.*
- 3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. Drawings need not show details, but should show the mathematics in the problem.
- 3.OA.4 4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = \div 3$, $6 \times 6 = ?$*

Understand properties of multiplication and the relationship between multiplication and division.

- 3.OA.5 Apply properties of operations as strategies to multiply and divide. *Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known, (Commutative property of multiplication) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)* Students need not use formal terms for these properties.
- 3.OA.6 Understand division as an unknown-factor problem. *For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.*

MEASUREMENT AND DATA (MD)**Geometric measurement: understand concepts of area and relate area to multiplication and to addition.**

- 3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.
- A square with side length 1 unit called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
 - A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.
- 3.MD.7 Relate area by operations of multiplication and addition.
- Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
 - Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

MEASUREMENT AND DATA (MD) Cont.

- c. Use tiling to show in a concrete case that the area of a rectangle with whole number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$ (represent the distributive property with visual models including an area model).
- d. Recognize area as additive. Find the area of figures composed of rectangles by decomposing into non-overlapping rectangles and adding the areas of non-overlapping parts, applying this technique to solve real-world problems.

Commentary:

The major work of the Operations and Algebraic Thinking domain in Grade 3 is to develop students' conceptual understanding of multiplication and division by using concrete materials to model multiplication and then relate their understanding of multiplication to division. The relationship between multiplication and division helps students understand that when dividing, they are finding the number of groups (missing factor) when they know the total count (product) and the number of items in a group (factor), or finding the number of items in a group (missing factor) when they know the number of groups (factor) and the total count (product). Problem solving situations and activities that include a variety of representations showing equal-sized groups, arrays, and area models lay the foundation for multiplication and division of whole numbers.

Note that these Standards are not linear. It is important for students to understand the meaning of multiplication and division (3.OA.1, 3.OA.2) through the use of problem situations (3.OA.3). As students demonstrate understanding they begin to relate models to symbolic notation (3.OA.4). The use of symbols for easier facts and relating the symbols to fact families should be happening as students continue to use models to solve problems with the more difficult facts.

As students have a variety of experiences solving problems and modeling multiplication and division situations with one-digit factors, they explore the properties of multiplication, develop strategies based on these properties, and use the properties to build their understanding of the relationship between multiplication and division. Properties include the commutative and associative properties, the identity element for multiplication, and the zero property. These properties can be connected to earlier work with addition. The distributive property will help students develop efficient strategies for multiplication-not only or basic facts but also for more complex multiplication examples. It is also foundational property for future work with algebra.

Third graders will recognize area as an attribute of two-dimensional regions. Students will measure the area of a shape by finding the number of square units needed to cover the shape. Students will learn that rectangular arrays can be decomposed into identical rows or into identical columns. Students will also connect the concept of area to multiplication by decomposing rectangles into rectangular arrays of squares.

Instructional Resources

Math Expressions: Unit 1 (Sections: 1.1 – 1.19); Unit 2 (Sections 2.1 – 2.8)

Manipulatives: Student Math White Boards, Hundreds Chart, two-color counters, grid paper, pattern blocks, plastic cups, fluency folders, Class Multiplication Table

Achieve The Core Fluency Resource

Digital: Think Central – Soar to Success for Below Level and Mega Math for On Level

Differentiated Instructional Activities: Pages: 9, 17, 31, 43, 53, 61, 71, 81, 87, 97, 107, 117, 125, 131, 143, 149, 155, 161, 167, 181, 191, 201, 209, 217, 225, 231 and 239

Math Literature Library: “A Grain of Rice” and “The Doorbell Rang”

Assessment Resources

Unit 1: Quick Quiz 1, 2, 3 and 4

Unit 1 Assessment Form A and Form B

Unit 2: Quick Quiz 1

Formative: “Check for Understanding embedded in each lesson.

Addressing Student Misconceptions and Common Errors**3.OA.1**

In previous work with addition, both addends represented the count or number of items that are joined for a total count. For example, 6 markers and 3 more markers give a total of 9 markers. In multiplication, one factor represents the number of groups, sets, or collections, and the other factor represents the number of items in each group, set or collection. Students need multiple experiences identifying which factor represents the number of groups and which factor represents the number of items in each group. Early experiences with concrete models and pictures and explicit connections to the symbolic notation will not only help students to identify multiplication situations but will also support student understanding of division.

Addressing Student Misconceptions and Common Errors**3.OA.2**

Because multiplication is commutative ($3 \times 7 = 7 \times 3$), some students think that $21 \div 3$ and $3 \div 21$ mean the same thing. This is especially true the equations are written two different ways.

$$21 \div 3 \text{ and } 3\sqrt{21}$$

Connecting concrete and pictorial models to both forms of division equations is essential to eliminating the misconception.

Students read $3\sqrt{21}$ as 3 “goes into” 21. Although these words are commonly used, they do not reinforce the meaning of division. Getting students to read this as “3 divides 21” or “21 divided by 3” or “How many groups of 3 are in 21?” is a habit that should be developed early in division instruction.

The sharing model (How many in a group?) is often easier for students to recognize s division. The measurement model is more difficult. Students need to work with many problem situations for each type of division using concrete materials and drawing pictures.

3.OA.3

Students who have trouble identifying information in a problem situation (which number represents the total, the number of groups and/or the number of items in a group) need more experience making explicit connections between their representations (concrete models or pictures) and determining the number of groups or the number of items in a group.

3.OA.4

Now that students are working more frequently with numeric equations for multiplication and division, reinforce accurate reading of the equations. $15 \div 3$ and $3\sqrt{15}$ should be read as “15 divided by 3” or “3 divides 15”. In algebra, the use of a symbol in one problem cannot represent a different number in a different but related situation. When writing missing factor equations, be sure to use different symbols for the missing factor that represent the number of groups and the missing factor that represents the total number in a group or the total number of items. For example: $15 \div 3 = x$ $15 \div 5 = y$

Addressing Student Misconceptions and Common Errors Cont.**3.OA.5**

Students often confuse multiplying by zero with adding to zero. Although this property seems obvious, providing students with problems and using models will help to reinforce the correct understanding.

The distributive property forms the foundation for all future work with multiplying whole number. However, in Grade 3, students should use this valuable property to help learn more difficult basic facts through array models. Introduce and continue work with this property throughout early work with multiplication. Students need opportunities to use and describe this property in order to make sense of it.

3.OA.6

Students often consider multiplication and division as discrete operations and do not understand the importance of the relationship between them as they learn basic facts to solve problems. It is important for students to understand division in terms of finding a missing factor and relate this work to writing division expressions and equations. Students need much experience identifying what information is known and what they are looking for using concrete materials and drawing pictures as well as asking themselves the right question, such as “How many groups of 7 can I make from 28?” Relating work with models to written missing factor multiplication equations and division equations is essential for students to develop this understanding.

3.MD.5

Students may incorrectly miscount the unit squares covered to determine the area of a shape using graph paper. To avoid an incorrect count, students can put the numbers of the counting sequence in each as they count them. When students use geoboards to create very unusual shapes, they may not be able to determine the area with square units. Help students visualize square units as they use geoboards to find the area.

3.MD.7

Instead of multiplying, some students may merely count unit squares to determine the area. Applying multiplication facts may be an issue. To address this, have them sketch a rectangle with rows of squares and ask them to write a number sentence instead of counting, such as “4 rows of 5 squares = $5 + 5 + 5 + 5 = 4 \times 5 = 20$ squares.”

Addressing Student Misconceptions and Common Errors Cont.**3.MD.7**

Some students may count unit squares to determine the area without realizing that the distributive property with multiplication may make the area of rectangular region easier to find. To address this, teacher can create additional experience with tiles to determine area using the distributive property. Students should describe and explain how they found the area.

Some students may be challenged by simply visualizing and finding the rectangles in the figures. Provide additional experiences for these students to locate the rectangles before finding the area.

Source: The Common Core Mathematics Companion: The Standards Decoded (What They Say, What They Mean, How to Teach Them)

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